



DIKLAT GURU PENGEMBANG MATEMATIKA SMK JENJANG LANJUT TAHUN 2009

BAHASA INGGRIS DALAM PEMBELAJARAN MATEMATIKA



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DIREKTORAT JENDERAL PENINGKATAN MUTU PENDIDIK DAN TENAGA KEPENDIDIKAN
PUSAT PENGEMBANGAN DAN PEMBERDAYAAN PENDIDIK
DAN TENAGA KEPENDIDIKAN MATEMATIKA

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KATA PENGANTAR

Puji syukur kami panjatkan ke hadirat Tuhan Yang Maha Esa, karena atas karunia-Nya, bahan ajar ini dapat diselesaikan dengan baik. Bahan ajar ini digunakan pada Diklat Guru Pengembang Matematika SMK Jenjang Lanjut Tahun 2009, pola 120 jam yang diselenggarakan oleh PPPPTK Matematika Yogyakarta.

Bahan ajar ini diharapkan dapat menjadi salah satu rujukan dalam usaha peningkatan mutu pengelolaan pembelajaran matematika di sekolah serta dapat dipelajari secara mandiri oleh peserta diklat di dalam maupun di luar kegiatan diklat.

Diharapkan dengan mempelajari bahan ajar ini, peserta diklat dapat menambah wawasan dan pengetahuan sehingga dapat mengadakan refleksi sejauh mana pemahaman terhadap mata diklat yang sedang/telah diikuti.

Kami mengucapkan terima kasih kepada berbagai pihak yang telah berpartisipasi dalam proses penyusunan bahan ajar ini. Kepada para pemerhati dan pelaku pendidikan, kami berharap bahan ajar ini dapat dimanfaatkan dengan baik guna peningkatan mutu pembelajaran matematika di negeri ini.

Demi perbaikan bahan ajar ini, kami mengharapkan adanya saran untuk penyempurnaan bahan ajar ini di masa yang akan datang. Saran dapat disampaikan kepada kami di PPPPTK Matematika dengan alamat: Jl. Kaliurang KM. 6, Sambisari, Condongcatur, Depok, Sleman, DIY, Kotak Pos 31 YK-BS Yogyakarta 55281. Telepon (0274) 881717, 885725, Fax. (0274) 885752. email: p4tkmatematika@yahoo.com

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KOMPETENSI

Mampu memahami literatur berbahasa Inggris sederhana dan mampu menulis naskah sederhana berbahasa Inggris.

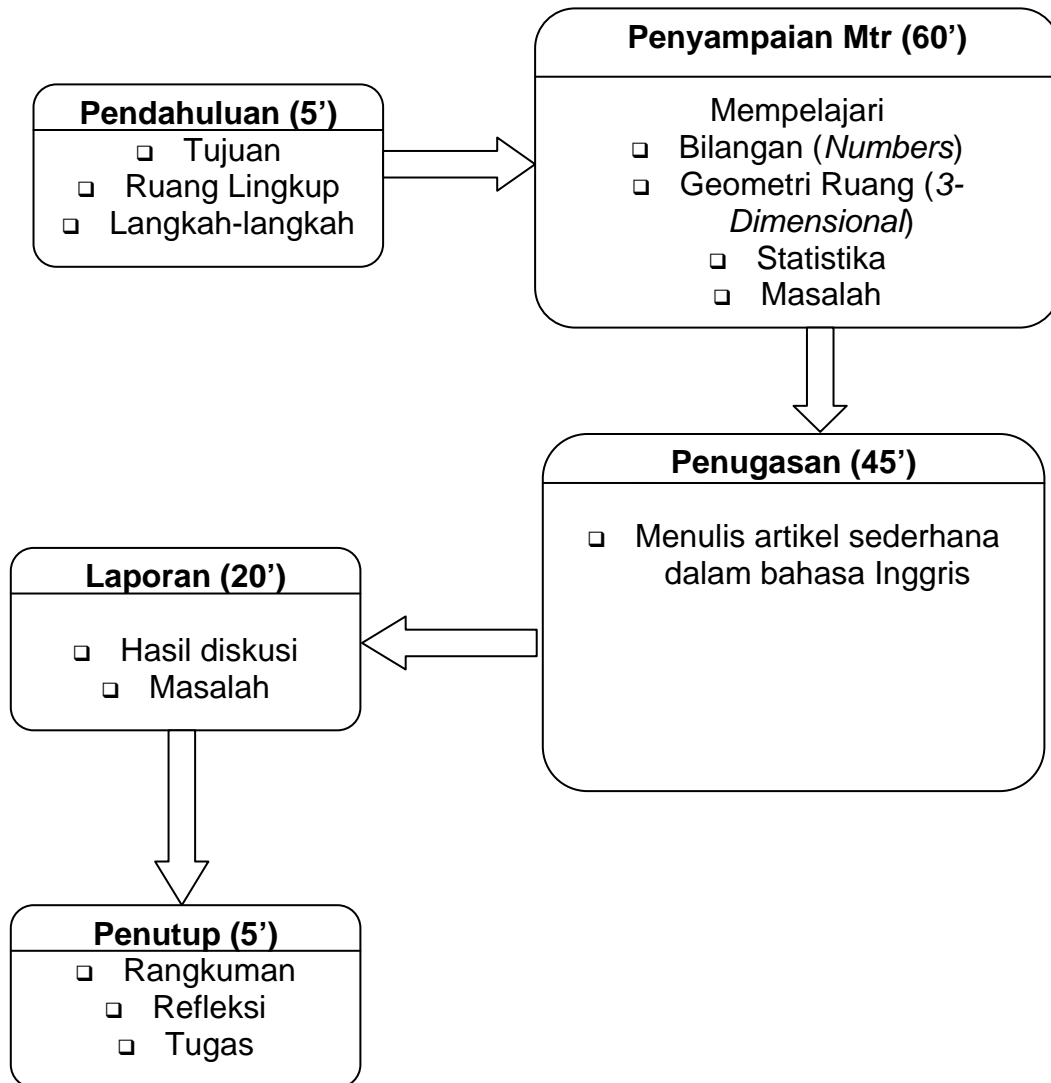
SUB KOMPETENSI

- Memiliki kemampuan membaca dan menjelaskan hal-hal yang berkait dengan Aljabar dalam bahasa Inggris.
- Memiliki kemampuan membaca dan menjelaskan hal-hal yang berkait dengan geometri dimensi tiga dalam bahasa Inggris.
- Memiliki kemampuan membaca dan menjelaskan hal-hal yang berkait dengan statistika dalam bahasa Inggris.
- Memiliki kemampuan memecahkan masalah dalam bahasa Inggris.
- Memiliki kemampuan menulis naskah sederhana dalam bahasa Inggris.

PETA BAHAN AJAR

Mata diklat untuk lanjut ini membutuhkan pengetahuan prasyarat berupa pengetahuan bahasa Inggris jenjang dasar yang sudah diberikan pada diklat jenjang dasar. Pada diklat jenjang lanjut ini kepada para peserta diberikan pengetahuan lanjutan yang berkait dengan aljabar, geometri dimensi tiga (geometri ruang), statistika, dan masalah dalam bahasa Inggris.

SKENARIO PEMBELAJARAN



Unit I

Introduction

A. Rationale

Mathematics is very important. Under the title '*Why teach mathematics*'; Cockroft report (1986: 1) stated that: "*We believe that all these perceptions of the usefulness of mathematics arise from the fact that mathematics provides a means of communication which is powerful, concise, and unambiguous.*" Cockroft report stated that Mathematics can be seen as language.

English is very important too, not only for students of SMK (Secondary Vocational School); but also for mathematics teacher of Secondary Vocational School. The reason is, there is a lot of mathematics or mathematics education books, periodicals, journals, video cassette recorders, or films, are written or talked in English.

During the learning of language, such as Indonesian and English; I personally believe that I learn by reading and hearing. When I met a new word (although in Indonesian language), then I tried to relate to the word before or after that word contextually. I personally support to use bilingual (Indonesian and English) during the teaching and learning mathematics, especially for 'high ability students' in order to ensure that Indonesian students can learn both Indonesian and English in schools.

In addition, some of Secondary Vocational School has been or will be declared to be Schools Based International (*Sekolah Berwawasan Internasional = SBI*). In those schools, English is used or will be used during the teaching and learning process. In anticipating this situation, mathematics teacher should be proficient in english, written or orally. To fulfill those needs, during the inservice training, one of the topic is 'Bahasa Inggris dalam Pembelajaran Matematika' or 'English in Teaching and Learning Mathematics.' This materials will be used during the session.

B. Objectives

The general aim of the session is to help mathematics teacher to understand english literature especially in mathematics and mathematics education literature. After the session, the participant will be able to:

- Read and explain the mathematics materials concerning terms in Algebra.
- Read and explain the mathematics materials concerning terms in 3-Dimensional Geometry
- Read and explain the mathematics materials concerning terms in Statistics
- Read mathematical problems; solve and write the answers in English.

C. The Used of Materials

In this module, the materials are written in such a way that can be learned by participants by themselves. Ideally, the materials can be learned before the session. During the session, participants can ask to the tutor (*Widyaiswara*) about those materials, especially the spoken problems.

Unit 2 Algebra

A. Powers and Roots

When we write 4^2 (four **squared**), or x^2 (x **squared**), the 2 is called the **power** or **index**.

a^2 means $a \times a$.

$a \times a^2$ (a times a squared) is equal to a^3 (a **cubed**).

$a^2 \times a^3$ (a squared times a cubed) is equal to a^5 (a **to the power of five** or a **to the fifth** (power)).

In this example we simply add the indices or powers.

$(b^2)^3$ (b squared **all cubed**) is equal to b^6 (b **to the power of six**).

This example shows that to **rise a power to a power**, we multiply the indices.

$\sqrt{64}$ means the **square root** of 64.

$\sqrt[3]{27}$ means the **cube root** of 27.

$\sqrt[5]{x}$ means the **fifth root** of x.

If we wish to find the root in, for example: $\sqrt[n]{a^m}$, we must divide the index by the root.

$$\sqrt[n]{a^m} = a^{m:n} = a^{\frac{m}{n}}$$

$\sqrt[3]{a^2}$ is equal to $a^{\frac{2}{3}}$. Here, the index ($\frac{2}{3}$) is a fraction and is therefore called a **fractional**

a^2 index.

$a^2 : a^4 = a^{-2}$ (**a to the [power of] negative 2**), which is called a **negative index**.

Practice 1.A

1. Read out the following:

a. $a^2 + b^2$

b. $x^2 + y^3$

c. p^4

d. x^7

e. $4b^2 \times 2b^3 = \dots$

f. $3y^2 \times y = \dots$

g. $x^5 : x^3 = \dots$

h. $z^n = (x + y)^2$

i. $6a^2 - 3a^2 = \dots$

j. $6y^2 : 3y^4 = \dots$

2. Read out the following and express them in more simple terms:

a. $\sqrt{x^2}$

b. $\sqrt{4x^4}$

c. $\sqrt{a^2b^2}$

d. $\sqrt[4]{m^4n^8}$

e. $\sqrt[3]{b^3}$

f. $\sqrt[5]{a^{30}}$

g. $\sqrt[3]{8a^6}$

h. $\sqrt[3]{\frac{1}{a^{12}}}$

i. $\sqrt[4]{\frac{x^{12}}{y^{-8}}}$

j. $\sqrt[2]{25x^{-4}y^{10}}$

3. Read out the following and say what their value is:

a. 2^3

b. 3^2

c. $9^{\frac{1}{2}}$

d. $8^{\frac{2}{3}}$

e. $25^{\frac{1}{2}}$

f. 2^{-3}

g. 5^{-1}

h. 3^0

i. $8^{\frac{2}{3}}$

j. $32^{\frac{3}{5}}$

4. Fill in the blank spaces in the following sentences:

- Any number (except 0) to the ---- of 0 (nought) is equal to ---- .
- To divide powers we ---- the ---- .
- To ---- a ---- to a power, we ---- the indices.
- a to the ---- of five divided ---- a ---- equals a cubed.
- The ---- ---- of forty-nine is seven.

B. Factors

If one number divides exactly into a second number, the first is a **factor** of the second, and the second is a **multiple** of the first.

The fraction $\frac{8}{24}$ is normally written as $\frac{1}{3}$. It is normal to express fraction **in their lowest terms**. Here 2, 4, and 8 are all factors of both the numerator and the denominator, but 8 is the **highest common factor** (HCF). A factor which is also a **prime number** (2, 3, 5, 7, 11, etc) is called a **prime factor**.

The smallest number which is exactly **divisible** by two or more numbers is called their **lowest common multiple** (LCM). The LCM of 24 and 36 is 72.

If the expression $3x(3x - 5)$ is **expanded**, we obtain the result $9x^2 - 15x$.

If the expression $9x^2 - 15x$ is **factorised**, we reverse the process and obtain the result $3x(3x - 5)$.

In an expression where it is difficult to discover the HCF, it helps to **group the terms**. For example, factorise the following expression:

$$ax - ay + bx - by$$

a is a factor of the first two terms, and b is a factor of the second two terms. Thus, by grouping the terms, we obtain:

$$a(x - y) + b(x - y).$$

Here $(x - y)$ is a common factor, so we factorise again to obtain the result:

$$(x - y)(a + b).$$

If an algebraic expression is made up of two terms, e.g. $(x + 3)$ or $(2y - 4)$; it is called a **binomial**. An algebraic that is made up of three terms, e.g. $(2x^3 + 3x^2 - 4)$ is called a **trinomial**.

A trinomial is the product of two binomials; e.g. $(a + 5)(a - 2) = a^2 + 3a - 10$ and so the factors of a trinomial can be expressed as two binomials.

Practice 1.B

1. Write down the answers to the following:
 - a. What are the prime factors of thirty-eight?
 - b. What is the highest common factor of eighteen and twenty-six?
 - c. What is the lowest common multiple of six and eight?
 - d. Express the fraction fourteen over twenty-one in its lowest terms.
2. Expand the following:
 - a. Three x minus four all squared.
 - b. Two y plus nine all squared.
 - c. Five a minus four all squared.
 - d. Four plus 2 r all squared.
 - e. Four p minus two q all squared.
3. Fill in the blank spaces in the following sentences:
 - a. Twenty-three has only two ---- , itself and ---- , and is therefore a ---- number.
 - b. To express a fraction or algebraic expression in its ---- ---- , we must divide its terms by the ---- ---- ----.
 - c. The factors of a ---- can be expressed as the ---- of two binomials.
 - d. Twenty-four is the ---- ---- ---- of twelve and eight.

C. Equations

If we wish to **solve** an equation, we must find the value of the letter (usually x) which **satisfies** the equation. When the equation is solved, the answer must be **checked**, by **substituting** it for **x in the original**.

Example.

$$x + 9 = 23$$

Here we subtract 9 from each side;

$$x + 9 - 9 = 23 - 9$$

Therefore $x = 14$.

We can check by substituting 14 for x .

Equations which we solve at the same time in order to find two unknown values are called **simultaneous equations**.

An expression which contains a square as the highest power of any letter (x^2 , y^2 , etc) is called a **quadratic**. If we say that such an expression is equal to some value, the resulting equation is known as a **quadratic equation**.

Example.

Solve: $x^2 - 3x - 4 = 0$

By factorising we get:

$$(x - 4)(x + 1) = 0$$

Therefore, either $x - 4 = 0$ or $x + 1 = 0$

So, $x = -1$ or $x = 4$.

$\therefore x = -1$ or 4

These values for x are the **roots** of the equation.

Practice 1.C

1. Find the number when seven times the number is four less than sixty-seven.
2. Find the number when twenty-eight is one more than three quarters of the number.
3. Find the number when five, plus three times the number, equals forty-one.
4. Three consecutive odd numbers add up to twenty-seven. What are they.
5. Two consecutive even numbers add up to thirty. What are they.

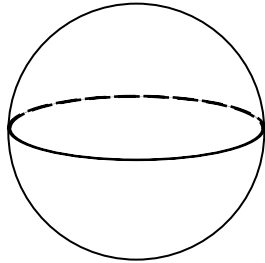
6. Solve the following:
 - a. b plus eight equals eleven
 - b. seven b equals forty-two
 - c. two x equals one
 - d. three y plus nine equals twenty-seven
 - e. four y minus eleven equals y plus one
 - f. seven b equals sixteen minus three b
 - g. five c plus six equals two c plus twenty-four
 - h. three x plus five equals two x

7. Fill in the blank spaces in the following sentences:
 - a. When we have ---- an equation, we should ---- our answer.
 - b. The answer is checked by ---- it for the letter in the original equation.
 - c. If our answer ---- the equation, it is correct.
 - d. We can use one equation to help us solve another equation. These are called ---- equations.
 - e. ---- are fixed equation which can be applied in certain regular situations.

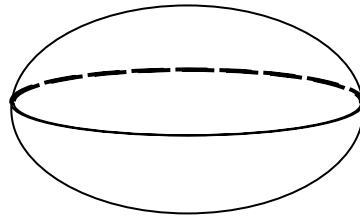
Unit 2

3-Dimensional Figures

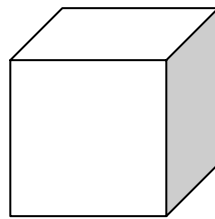
A. The following figures are **3-dimensional**.



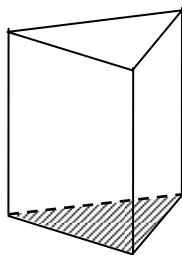
This is a **sphere**. Objects shaped like a sphere are **spherical**.



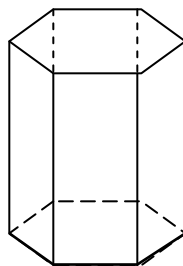
This is an **ellipsoid**. Objects with this shape are **ellipsoid**



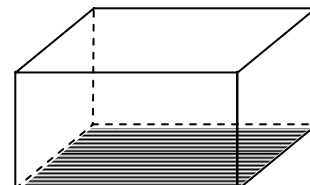
This is a **cube**. Objects shaped like a cube are **cubic**.



(a)

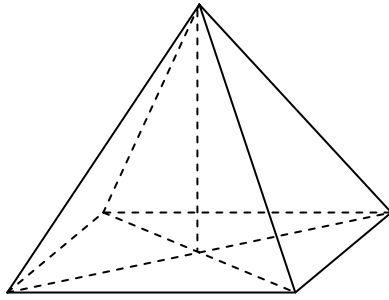


(b)

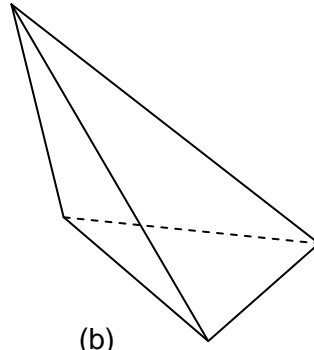


(c)

These shapes are **prisms**.
(a) is a **triangular prism**
(b) is a **hexagonal prism**
(c) is a **rectangular prism**

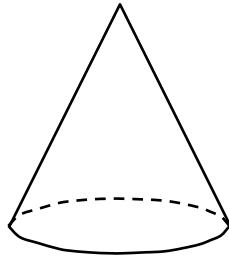


(a)

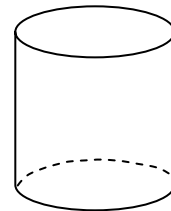


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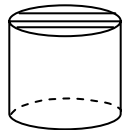
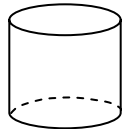
These shapes are **pyramids**.
(a) is a **right square pyramid**.
(b) is an **oblique triangular pyramid**.
Objects shaped like pyramids are **pyramidal**.



This shape is a **right cone**.
Objects shaped like a cone
are **conical**.



This is a **cylinder**.
Objects shaped like a cylinder
are **cylindrical**.



(a)

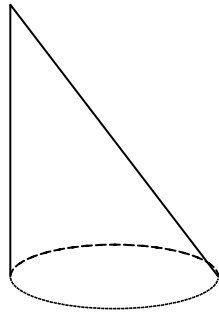


(b)

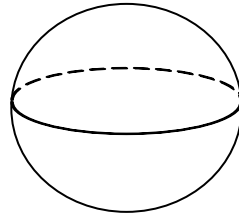
These figures show **cross-section** of a cylinder.
(a) is a **transverse** section (circular).
(b) is a **longitudinal** section (rectangular).

Practice 2.A

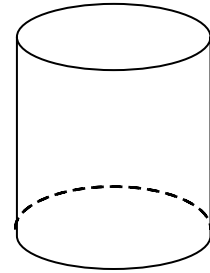
1. Identify the following shapes and their sections:



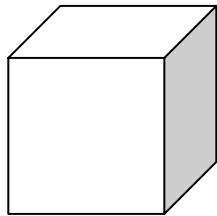
(a)



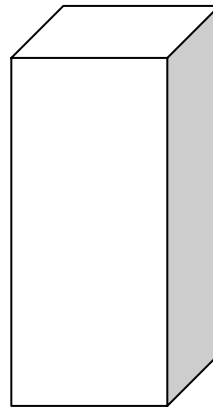
(b)



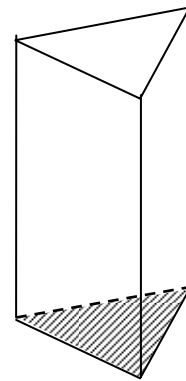
(c)



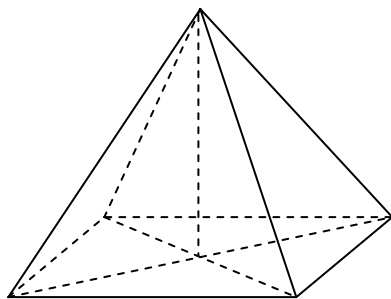
(d)



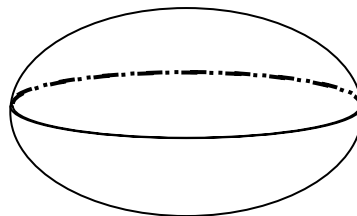
(e)



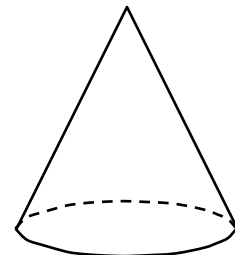
(g)



(h)



(i)

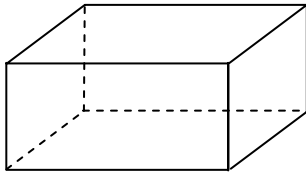


(j)

2. Draw the following sections.
- a. An oblique section of a cone.
 - b. A longitudinal section of a right pyramid.
 - c. A transverse section of a square pyramid.
 - d. A longitudinal section of a cylinder.
 - e. A transverse section of a square ellipsoid.

- f. A transverse section of a right hexagonal prism.
- g. A longitudinal section of a right cone.
- h. A longitudinal section of a right triangular pyramid.
- i. A transverse section of a right triangular pyramid.
- j. An oblique section of a cylinder.

B. Dimensions



This rectangular solid has a length of 5m.

It is 5m long.

It has a width of 3,5m.

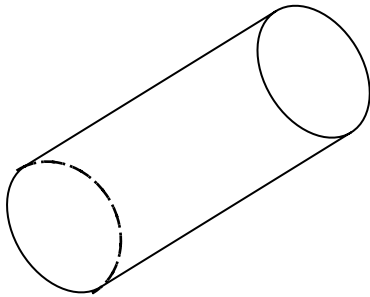
It is 3,5m wide.

It has a height of 1m.

It is 1m high.

It has a surface area of 52m² (square metres). ($A = 2hw + 2hl + 2lw$).

It has a volume of 17,5m³ (cubic metres). ($V = lhw$)



This cylinder has a length of 65cm.

It is 65cm long.

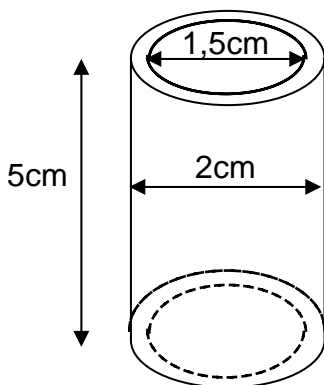
It has a radius of 10cm.

It has a diameter of 20cm.

It has a surface area of 4714cm² (square centimetres). ($A = \pi dh + 2\pi r^2$).

It has a volume of 20428cm³ (cubic centimetres). ($V = \pi r^2 h$).

$$\pi = \frac{22}{7}$$



This tube has an **external diameter** of 2cm.

It has an **internal diameter** of 1,5cm.

It has a **bore** of 1,5cm.

The wall of the tube is 0,25cm **thick**.

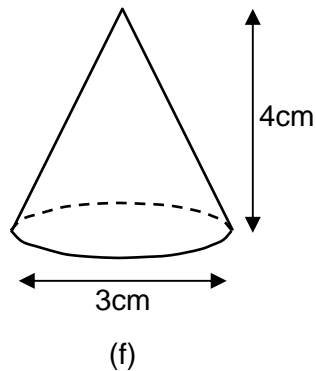
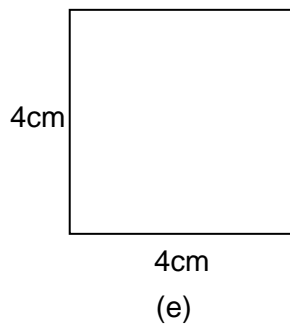
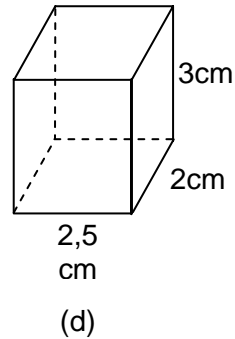
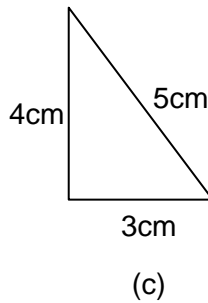
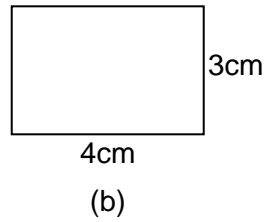
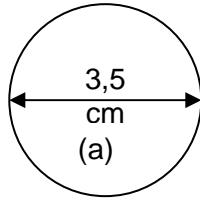
It has a **thickness** of 0,25cm.

It has a surface area of ... cm² (square centimetres). ($A = \pi d_1 h + \pi d_2 h + 2(\pi r_1^2 - \pi r_2^2)$).

It has a capacity of ... cm³ (cubic centimetres). ($C = \pi r_2^2 h$).

Practice 2.B

1. Describe the following figures. The measurement are given

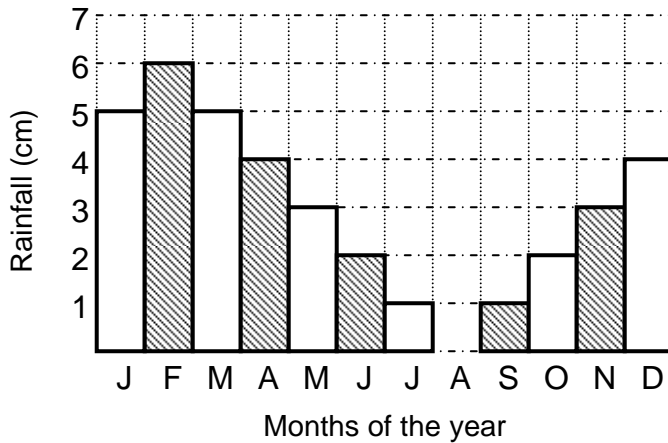


2. A table has a length of 1 metre and a width of 80cm. The top of the table is 4cm thick. The overall height of the table is 64cm. The legs of the table measure 60×6×5. Draw a side view of the table on the scale 1:20.
3. A clock has a circular face with a diameter of 20cm. The minute hand is 8cm long and the hour hand is 5cm long. The case of clock is rectangular with a length of 30cm and a height of 25cm. Draw a frontal view of the clock on the scale 1:2 with the clock showing the time at 10 minutes past 10.
4. A country's national flag is 180cm wide and 100cm deep. It consists of three equal vertical stripes: blue, green, and black from left to right. In the central position there is white disc with a diameter of 50cm. Draw a picture of this flag on the scale 1:10.

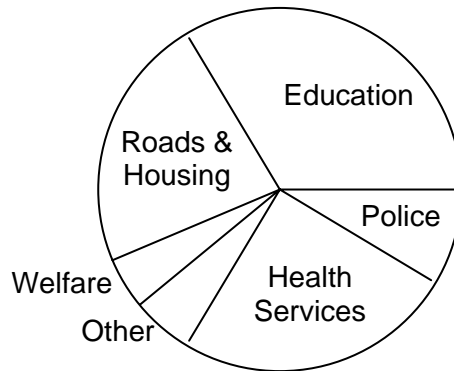
Unit 4 Statistics

A. Simple Graphs

Block diagram (or bar graphs, or histogram)

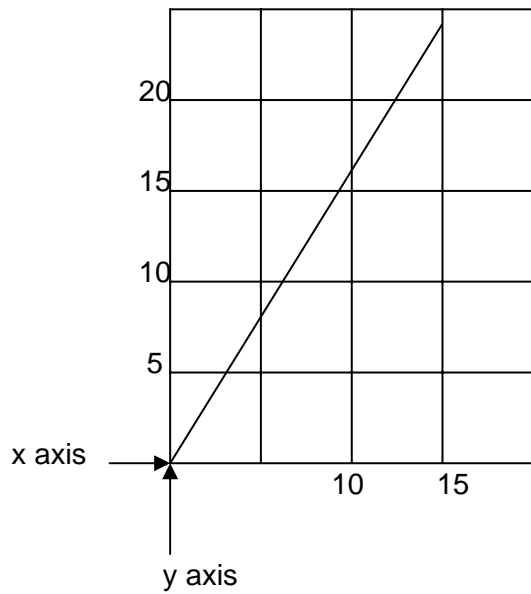


Circular (or 'pie') diagram

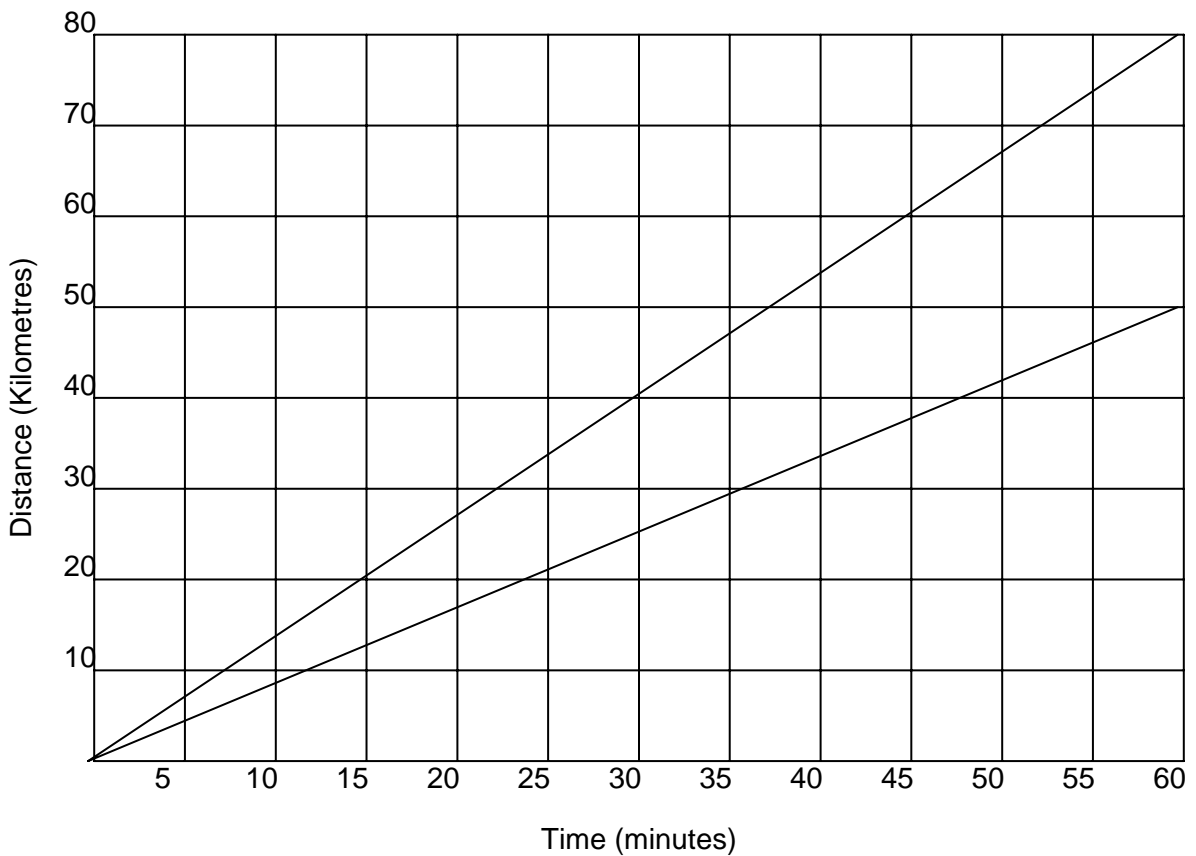


The figures above illustrate two types of diagram that are commonly used outside mathematics. The block diagram consists of a number of straight lines drawn to scale. In the circular diagram each sector represents a fraction of the whole quantity – in this case the money available for local services.

A graph in mathematics is a line which is **plotted** on to **axes**, the **x axis** and the **y axis**. This figure shows a **straight line graph** showing the **conversion rates** of dollars (US) and pounds sterling (1 pound sterling = \$1,65)



This figure shows another straight line graph. It compares two different speeds, and therefore the lines have two different **gradients**. The gradient is the ratio of the vertical rise to the horizontal distance.

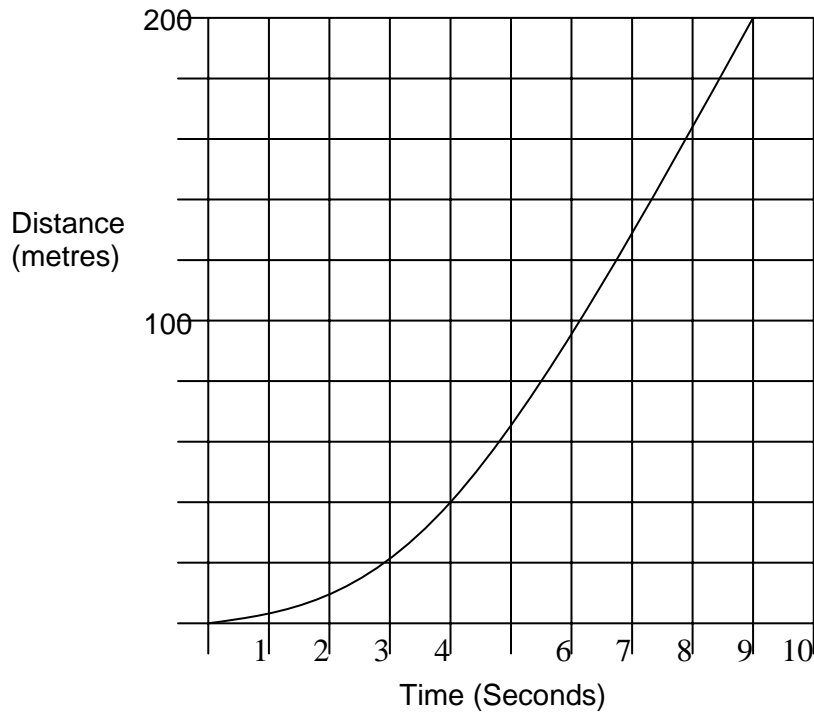


Practice 3.A

1. In the speed graph above, what quantities are plotted on the x and y axes?
2. What does the graph represent?
3. Why are the lines straight?
4. The graphs of quantities that do not vary in direct proportion are usually in the form of a **curve**. The formula:

$$S = \frac{1}{2} at^2$$

enables us to calculate how far a car will travel from rest in a given time at a particular rate of acceleration. In this formula, S = distance in metres; a = acceleration in metres per second per second; and t = time in second. The curve of the relationship among S, a, and t is:



- a. Which value is plotted on the x axis?
- b. Which value is plotted on the y axis?
- c. What is the rate of acceleration of the car whose journey is plotted on that graph? How can you obtain this information from the graph?
- d. Why the graph is curved?

Unit 5

Collection of Mathematical Problems

A problem was defined by Cooney, Davis & Henderson (1975) as: "... a question which presents a challenge that *cannot* be resolved by some *routine* procedure known to students."

Two types of problems have been identified (Charles, 1982; le Blanc, Proudfit & Putt, 1980): (1) standard textbook (translation) problems, and (2) process problems.

In solving *translation problem*, the emphasis is on translating a real word situation in the problem into mathematical terminology or mathematical sentence in the solution. The Translation problem requires only the application of skills, principles, or concepts known to students, while the *process problem* requires, in addition, the use of strategy or some non-algorithmic approach. Process problems emphasize the process of obtaining the solution rather than solution itself.

Shoenfeld (1985), in his book *Mathematical Problem Solving*, described four requirements for solving mathematical problems:

1. *Resources*. Mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand.
2. *Heuristic*. Strategies and techniques for making progress on unfamiliar or nonstandard problems; rules of thumb for effective problem solving
3. *Control*. Global decisions regarding the selection and implementation of resources and strategies.
4. *Belief Systems*. One's mathematical world view", the set of (not necessarily conscious) determinants of an individual's behavior.

Every students and teachers should learn and practice solving problem *translation problem* and *process problem*. In order to help our students, please solve the following questions or problems.

Practice 4.A

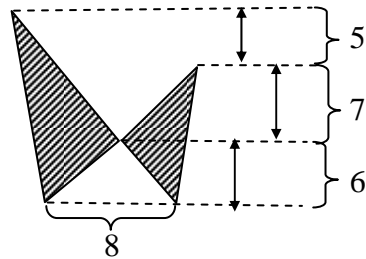
1. Now we are in the year 2003. The ratio of ages of my father, my mother, and my younger brother is 12 : 9 : 1. Five years from now, my father will be 41 years old. In what year my younger brother born?
2. Laila's savings in bank is \$100. Tina's savings is \$40. Every end of the week, laila withdraws \$3 from her saving. At the same time, Tina always deposit \$2,40 into her savings. After how many weeks will Laila's savings be \$6 less than Tina's savings?
3. The product of two positif integers is even, but not divisible by 4. Is their sum odd or even?
4. On the table, there are 6 coins each of the values \$5, \$10, and \$50. Deni takes \$75. The number of coins Deni takes is more than 5 but less than 9. Does Deni take all the three

types of coins? If not, which type does he not take?

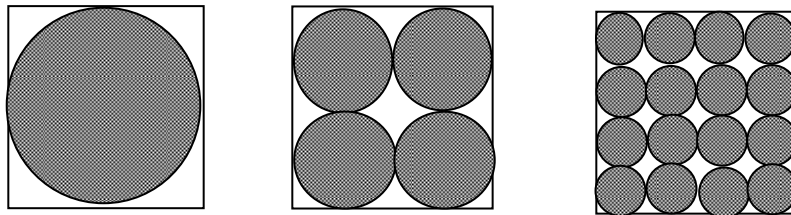
- The weight of a small box, two medium boxes and a large box altogether is 10 kg. The weight of the small box, two medium boxes and two large boxes altogether is 15 kg. What is the total weight of two small boxes and four medium boxes?
- Replace the letter A with an odd digit and the letter B with an even digit, so that 12 is a factor of the number A579B. Find the possible values of A579B.
- Find the missing digits.

$$\begin{array}{r}
 \square \square \square \\
 \square \square \\
 \hline
 2032 \\
 762 \\
 \hline
 9652
 \end{array} \times$$

- Find the total area of the shaded regions in the figure.



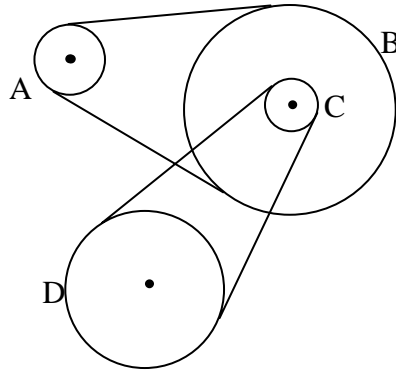
- In the following figures, the three squares have equal areas. Determine whether the areas of the three shaded regions in each square are also equal.



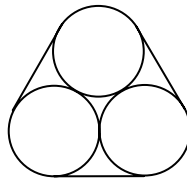
- The volume of a small balloon is 2 liters and a larger balloon is 5 liters. The small balloon is increased at rate of 0,3 liters per second. The larger balloon is decreased at rate of 0,12 liters per second. After how many seconds will the two balloons have the same volume?
- A train travel between two stasions. The train will be on time if it runs at an average speed of 60 km/hour, but will be late by 5 minutes if it runs at an average speed 50 km/hour.

What is the distance between the two stations?

12. Four wheels A, B, C, and D are connected by belts, see figure. The wheels B, and C are fastened together. The diameters of wheels A, B, C, and, D are 12 cm, 36cm, 9 cm and 27 cm, respectively. The wheel A turns at a speed of 450 rotations per minute. At what speed does the wheel D turn?



13. Three circular disks of radius 7 cm each are bound tightly with a belt, see figure. What is the length of the belt?



14. Twice the number of marbles in bag A is less than the number of marbles in bag B. The sum of the number of marbles in bag A and C is less than the number of marbles in bag B. There are more marbles in bag D than in bag B. There are 6 marbles in bag C and 9 marbles in bag D. How many marbles does bag B contain?

Unit 6 Concluding

Mathematics and English is very important, especially for students of SMK (Secondary Vocational School). This module hopefully can change the attitude toward English from the participant. This module has show you that English is not as difficult as your imagination. English can be learned easily.

Just as mentioned earlier on Unit 1, every students and teachers should be encouraged to learn English by trial and error, just as we learnt Indonesian when we were child. For mathematics teacher from Secondary Vocational School which has been or will be declared to be International Schools; you will have a lot of opportunities to learn English by learning Mathematics in English.

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